Factorising expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 – y^2$ is called the difference of two squares. It factorises to $(x – y)(x + y)$.

Examples

Example 1   Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2   Factorise $4x^2 – 25y^2$

$4x^2 – 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3   Factorise $x^2 + 3x – 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x – 10 = x^2 + 5x – 2x – 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) – 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x – 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4   Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5   Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \dfrac{x - 7}{2x + 3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

    a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

    c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

2 Factorise

    a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

    c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

    e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

    g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

    a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

    c $18a^2 - 200b^2c^2$

4 Factorise

    a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

    c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

    e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

    a $\dfrac{2x^2 + 4x}{x^2 - x}$ **b** $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$

    c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$ **d** $\dfrac{x^2 - 5x}{x^2 - 25}$

    e $\dfrac{x^2 - x - 12}{x^2 - 4x}$ **f** $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

    a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$ **b** $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

    c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$ **d** $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

1   **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

      c $5x^2y^2(5 - 2x + 3y)$

2   **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$

      c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$

      e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$

      g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3   **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$

      c $2(3a - 10bc)(3a + 10bc)$

4   **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$

      c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$

      e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5   **a** $\dfrac{2(x + 2)}{x - 1}$ **b** $\dfrac{x}{x - 1}$

      c $\dfrac{x + 2}{x}$ **d** $\dfrac{x}{x + 5}$

      e $\dfrac{x + 3}{x}$ **f** $\dfrac{x}{x - 5}$

6   **a** $\dfrac{3x + 4}{x + 7}$ **b** $\dfrac{2x + 3}{3x - 2}$

      c $\dfrac{2 - 5x}{2x - 3}$ **d** $\dfrac{3x + 1}{x + 4}$

7 $(x + 5)$

8 $\dfrac{4(x + 2)}{x - 2}$